Discrimination between statistical distributions for hydro-meteorological frequency modeling

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INTRODUCTION

- The identification of a statistical distribution to model the frequency of occurrence of extreme hydro-meteorological events is important.
- Two-parameter distributions such as the Generalized Pareto, lognormal, gamma or Weibull are useful in fitting datasets in areas such as POT extreme value modeling.
- Three-parameter distributions are also very important, such as for fitting annual maximum flood or precipitation series.

INTRODUCTION

- We recommend some methods of discrimination between distributions.
- The discriminations considered are between: Generalized Pareto (GP) and Kappa, Gumbel and some alternative frequency models, and model pairs belonging to the group {generalized extreme value (GEV), Pearson type 3 (P3), generalized logistic (GLO)}.
- Four discrimination methods are compared by Monte Carlo Simulation in terms of their discrimination power and discriminaton bias.

DISCRIMINATION STATISTICS

- Anderson-Darling statistic (AD)

$$A = A(X_n) = -n - \sum_{i=1}^{n} \frac{2i-1}{n}\left[\ln \hat{F}\left(X_{(i)}\right) + \ln\left\{1 - \hat{F}\left(X_{(n+1-i)}\right)\right\}\right]$$

Gives more weight to observations in the tails of the distribution as compared to Cramér-von Mises (CvM) and Kolmogorov-Smirnov (KS) statistics.

- Ratio of maximized likelihood statistic (RML)

$$T = T(X_n) = \ln\left[\frac{L\left(X_n; \hat{\theta}_0\right)}{L\left(X_n; \hat{\theta}_1\right)}\right]$$

Most widely investigated method.

DISCRIMINATION STATISTICS

- Transformation to normality followed by the application of Shapiro-Wilk GoF statistic (TN.SW)

$$Z_i = \phi^{-1}\left(\hat{F}(X_i)\right)$$

$$S = \left[\sum_{i=1}^{n} v_i Z_{(i)}\right]^2 / \sum_{i=1}^{n}\left[Z_{(i)} - \bar{Z}\right]$$

- Transformation to normality followed by the application of the Probability plot correlation coefficient statistic (TN.PPCC)

$$Z_i^* = \Phi^{-1}\left(\hat{F}(X_i)\right)$$

$$W_i^* = \Phi^{-1}\left(p_i\right)$$

DISCRIMINATION STATISTICS

The TN.PPCC statistic is then calculated as follows:

$$R^* = \frac{\sum\left(Z^*_{(i)} - \bar{Z}^*\right)\left(W^*_i - \bar{W}^*\right)}{\left[\sum\left(Z^*_{(i)} - \bar{Z}^*\right)^2 \sum\left(W^*_i - \bar{W}^*\right)^2\right]^{0.5}}$$

- Note: When the two frequency models have the same number of unknown parameters, applying RML is equivalent to using AIC or BIC.

PARAMETER ESTIMATION METHODS

- In practice, the parameters of the model are unknown, so they need to be estimated from the data.

- We considered three parameter estimation methods:

- Maximum likelihood (ML)
- Moments (MOM)
- Probability weighted moments (PWM).

DISCRIMINATION STUDIES

The discriminations considered are between:

- GP and KAP models
- Gumbel and some alternative frequency models
- Model pairs belonging to the group {GEV, P3, GLO):

  GEV vs P3, GEV vs GLO and P3 vs GLO.

GP AND KAP

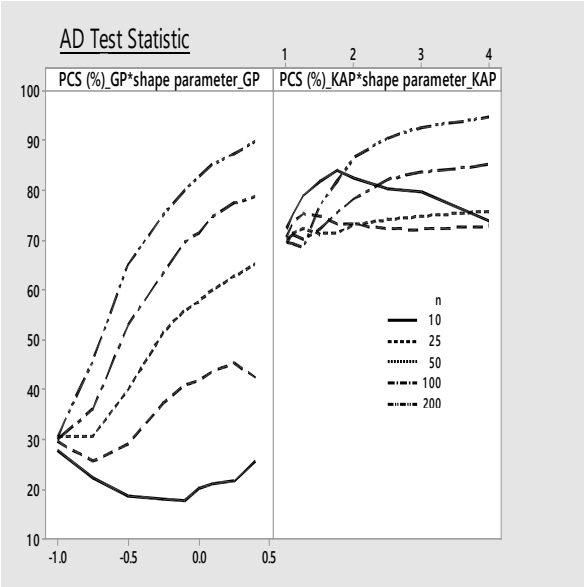

Fig. 1 PCS(%) using the AD statistic when GP is the true sampled distribution (left) and when KAP is the true sampled distribution (right).

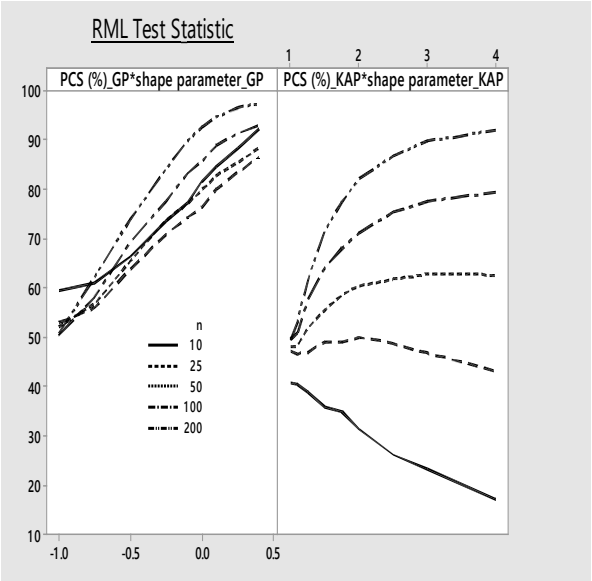

Fig. 2 PCS (%) using the RML statistic when GP is the true sampled distribution (left) and when KAP is the true sampled distribution (right).

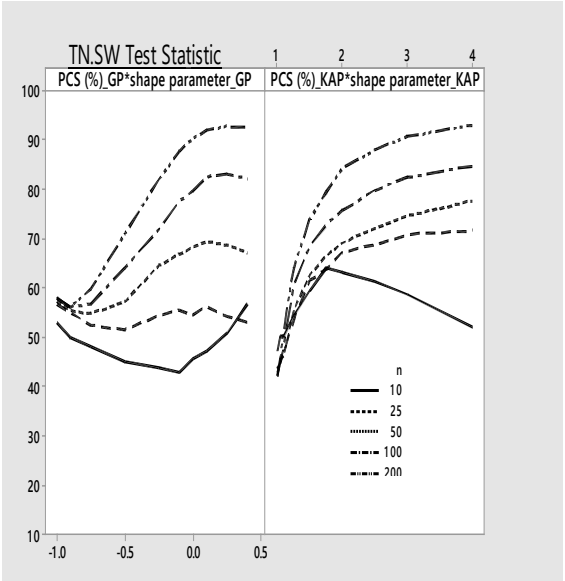

Fig. 3 PCS(%) using the TN.SW statistic when GP is the true sampled distribution (left) and when KAP is the true sampled distribution (right).

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GP AND KAP

Application with Eight Hydrological Datasets

Table 1. Selecting between the GP and KAP distribution for fitting POT flood data at eight hydrometric stations

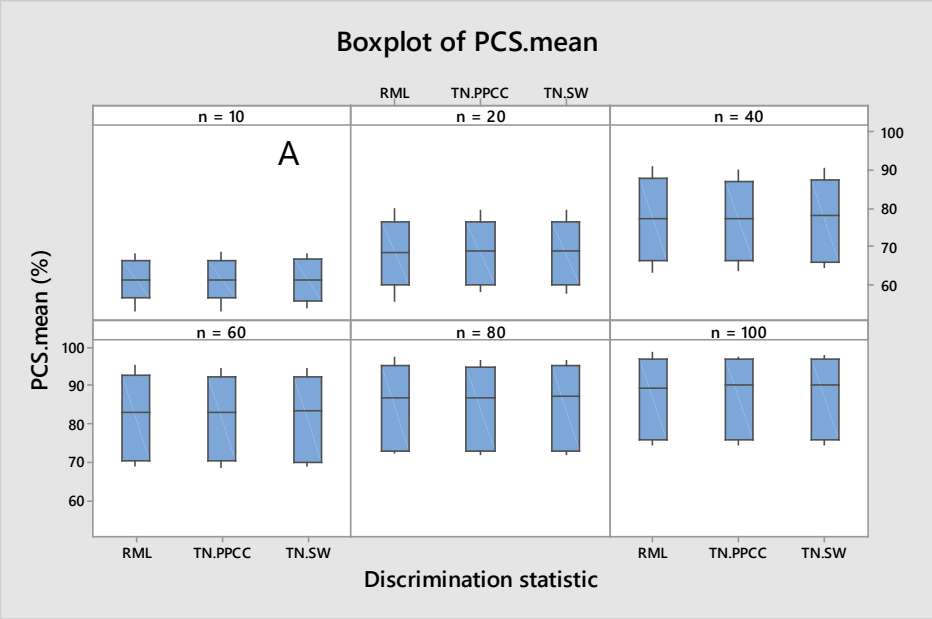
Station	n	GP Estimates (shape; scale)	KAP Estimates (shape; scale)	AD statistics (a_{GP}; a_{KAP})	TN.SW statistics (s_{GP}; s_{KAP})	RML statistics (t_{GP}; t_{KAP})
01AQ001	151	(-0.25; 19.98)	(1.95; 18.83)	(0.77; 0.23)	(0.989; 0.994) KAP is selected	(-1.88; 1.88) KAP is selected
01BL002	64	(0.01; 13.72)	(2.77; 12.89)	(1.06; 0.38)	(0.969; 0.982) KAP is selected	(-2.06; 2.06) KAP is selected
02FC002	159	(0.11; 119.5)	(2.80; 110.4)	(0.40; 1.08)	(0.992; 0.986) GP is selected	(3.03; -3.03) GP is selected
01BJ007	52	(-0.04; 430.8)	(2.20; 386.9)	(0.40; 0.61)	(0.983; 0.976) GP is selected	(1.05; -1.05) GP is selected
01AF007	47	(0.26; 28.49)	(4.00; 27.04)	(0.36; 0.24)	(0.985; 0.990) KAP is selected	(-0.33; 0.33) KAP is selected
04CA002	32	(0.03; 157.4)	(2.74; 156.2)	(1.00; 1.51)	(0.962; 0.959) GP is selected	(1.00; -1.00) GP is selected
02LB008	42	(-0.14; 41)	(1.92; 36.63)	(0.26; 0.41)	(0.977; 0.972) GP is selected	(0.81; -0.81) GP is selected
01BJ003	53	(0.13; 37.28)	(3.11; 35.65)	(0.27; 0.43)	(0.985; 0.987) KAP is selected	(0.52; -0.52) GP is selected

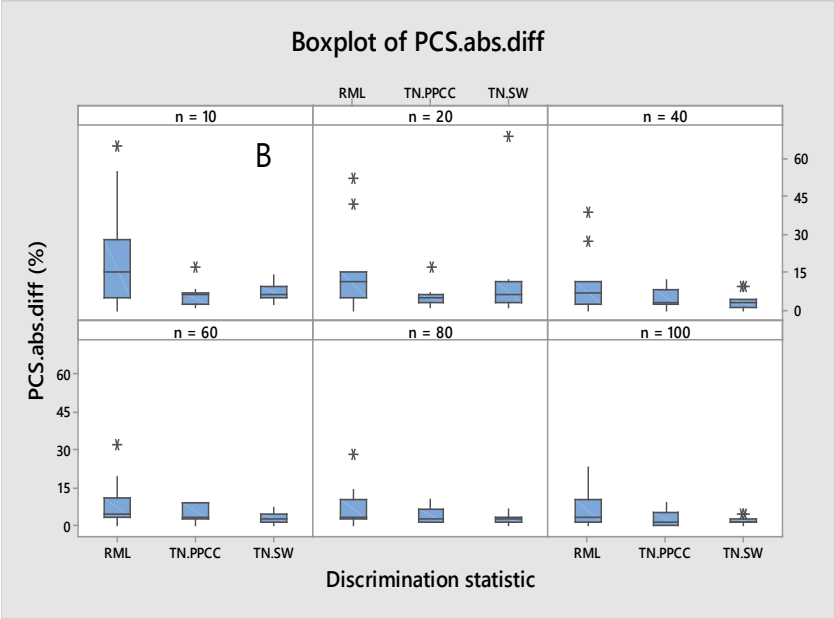
Gumbel and some alternative frequency models

The alternative models are:

- The normal: $N(\mu, \sigma)$
- The logistic: $LOG(\mu, \sigma)$
- Two student's t models to which a location parameter was added: $STU(\mu, \sigma, \nu)$
- Three models from the 3-parameter gamma family: $GAM3(\mu, \sigma, \gamma)$
- Four models from the GEV family: $GEV(\mu, \sigma, \gamma)$

Gumbel and some alternative frequency models





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GEV, P3 and GLO

Discrimination between GEV and GLO

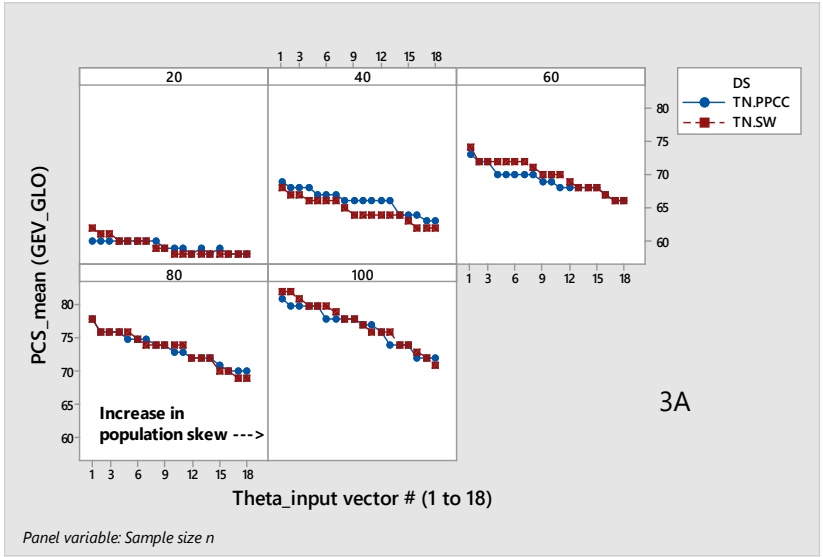

Fig. 5 PCS means for comparing TN.PPCC and TN.SW

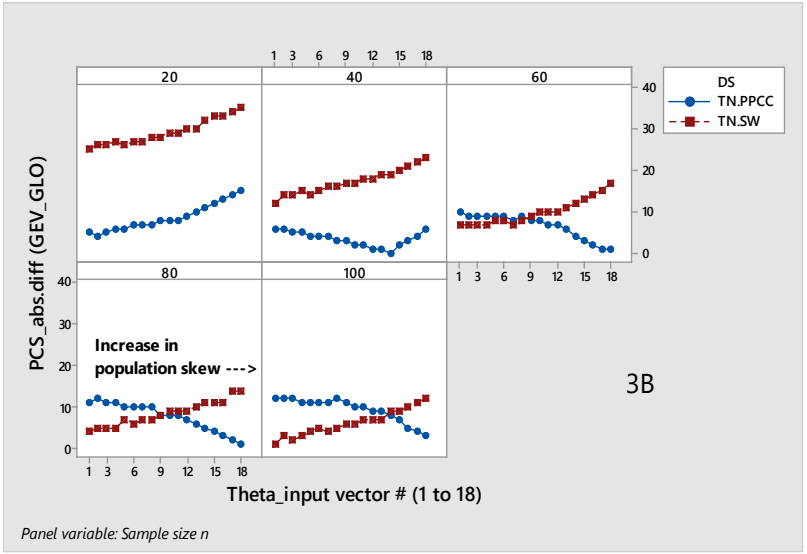

Fig. 6 PCS absolute difference for comparing TN.PPCC and TN.SW

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GEV, P3 and GLO

Discrimination between P3 and GLO

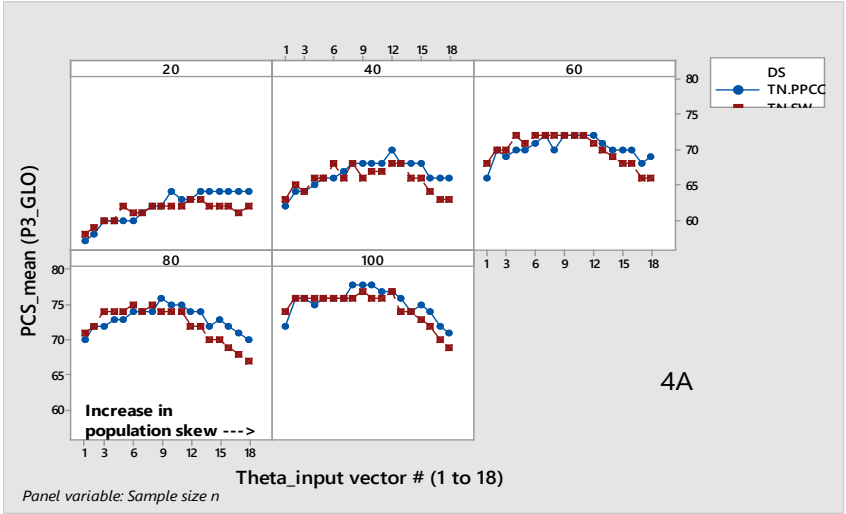

Fig. 7 PCS means for comparing TN.PPCC and TN.SW

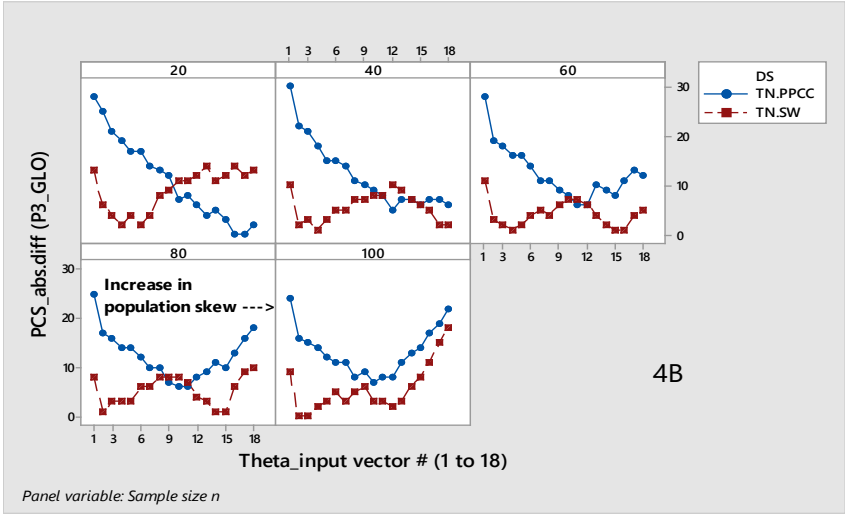

Fig. 8 PCS absolute difference for comparing TN.PPCC and TN.SW

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GEV, P3 and GLO

Discrimination between GEV and P3

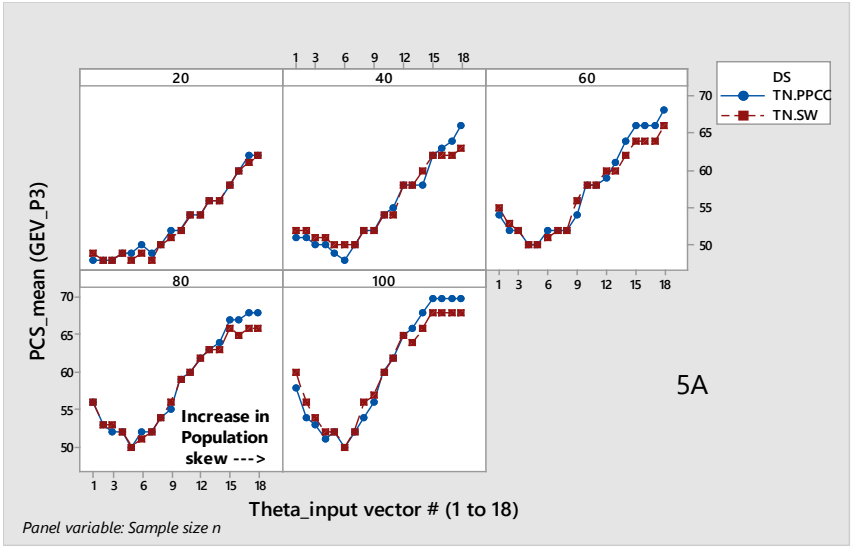

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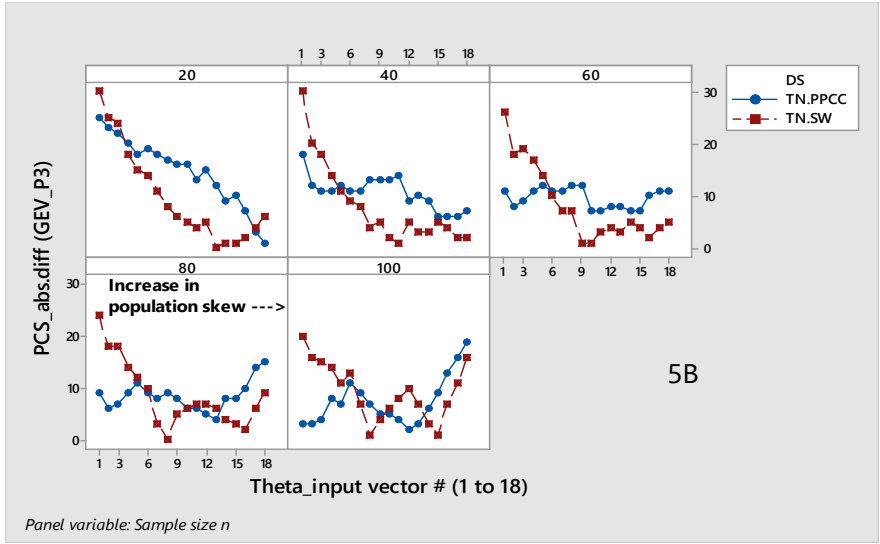
Fig. 9 PCS means for comparing TN.PPCC and TN.SW



5B

Fig. 10 PCS absolute difference for comparing TN.PPCC and TN.SW

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GEV, P3 and GLO

Table 2. Use of the TN.SW statistic s and the TN.PPCC statistic r^* to choose between the GEV, GLO and P3 models for the 18 data series

ID	Sample size	s.GEV	s.P3	s.GLO	r*.GEV	r*.P3	r*.GLO	Chosen.Model
#1	52	0.98111	0.97965	0.97082	0.99298	0.99238	0.98853	GEV
#2	37	0.97856	0.97838	0.96846	0.99072	0.99062	0.98689	GEV
#3	100	0.98867	0.98887	0.98667	0.99488	0.99521	0.99473	P3
#4	45	0.97498	0.97569	0.96414	0.99007	0.99037	0.98531	P3
#5	42	0.98632	0.98535	0.98497	0.99129	0.99066	0.99273	*
#6	32	0.97078	0.97264	0.96182	0.98952	0.99029	0.98557	P3
#7	43	0.98013	0.98252	0.97143	0.9921	0.99269	0.98851	P3
#8	96	0.98883	0.98994	0.98219	0.99493	0.99539	0.99234	P3
#9	78	0.98073	0.9801	0.98983	0.98965	0.9893	0.99549	GLO
#10	48	0.97653	0.98013	0.96938	0.99057	0.99197	0.98753	P3
#11	50	0.98684	0.97144	0.98792	0.99247	0.98284	0.99445	GLO
#12	41	0.9914	0.96647	0.98944	0.99473	0.97907	0.99521	*
#13	86	0.99426	0.97187	0.99101	0.99709	0.98388	0.99627	GEV
#14	52	0.98132	NA	0.97269	0.99247	NA	0.98861	GEV
#15	24	0.95023	0.97909	0.94225	0.97953	0.99168	0.97576	P3
#16	42	0.97923	0.97641	0.98231	0.98922	0.98713	0.99207	GLO
#17	20	0.94534	NA	0.93812	0.97752	NA	0.97403	GEV
#18	40	0.97626	NA	0.97174	0.99075	NA	0.98878	GEV

CONCLUSIONS

- To discriminate between the KAP and GP models, use of the AD statistic leads to bias for one model over the other.
- The use of RML for discriminating between three-parameter distributions led to some serious numerical problems.
- The TN.SW and TN.PPCC statistics proved to be the most advantageous among those considered, they would be recommendable in practice for this reason.
- We found a difficulty in discriminating between the P3 and GEV models.